H INFINITY CONTROL OF A MECHANICAL SYSTEM WITH BACKLASH

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ABSTRACT—*Backlash is a nonlinearity that occurs in a mechanical or a hydraulic system when two parts of that system are supposed to move together and there is an amount of space between the parts i.e.: Motor and Load Side. This non linearity is known to cause oscillations, inaccuracy and delays in the system. The performance of speed and position control is being affected by this non linearity. In this paper H infinity control approach has been selected to control the discretized model of the system with Backlash non linearity. Mechanical System operating in two modes, i.e. Contact Mode and Backlash mode. H infinity control design has been suggested to control the effect of this non linearity in the system. The robustness and good performance of the suggested control design has been compared with the performance of standard PID controller. By the comparison of results it has been found that the proposed controller is better and superior to PID controllers and other comparable linear controllers.*

Keywords - ***Backlash, H infinity control, Contact & Backlash Mode***

I INTRODUCTION

Backlash is a common problem which occurs whenever the transmission link is disconnected between the driving and driven side.e.g; the drive train in cars, robot Arm and printing presses. The full control of a load in the presence of backlash is a tough task when better and error free results are desired. The control of systems with backlash has been the subject of study since 1940s.The importance of taking the backlash into consideration in the power train model is identified by (De La Salle et al.1999). Mechanical solutions are also been proposed for the system with backlash such as spring loaded split gear assemblies and dual motor systems.Both are much capable to handle the problem mechanically, but they are expensive, energy consuming and more importantly increase the weight of the system. So the best approach is to achieve backlash compensation without such mechanical devices.

The H infinity control theory introduced by Zames is one of the advanced technique and with very wide and useful application in controls. H infinity control theory made a strong impact in the development of control systems in the decades of 1980 and 1990. H infinity techniques are of greater importance to deal with the multi variable systems. Whenever there is a need to minimize the closed loop impact of the disturbances, the H infinity control techniques used.

The research made on the H infinity control is classified for the state feedback system and for the output feedback system as well. At first, it is noted that in the H infinity control of a discrete time system , although the suitable controller exist for it but cannot be called as strictly suitable controller. The main reason of focusing on the proper strict controller is because in the non-strict controller, there is a possibility of lack of robustness. Research is also being made on the continuous plant with a discrete H infinity controller or the discrete plant with the discrete time controller. In this paper, a discrete time H infinity state Feedback controller has been designed for the discrete plant with the help of stabilizing solution of Discrete Algebraic Ricatti equation. The Discrete time H infinity control problem with strictly proper measurement feedback is discussed in [1]. Robust H2 and H∞ infinity controller of Discrete Time systems with polytopic uncertainties via Dynamic Output Feedback [2].

Previously, continuous time or Sampled PI controller was known to be the most suitable especially in case of two mass systems [3]. Low order H infinity Controller design (By an LMI Approach) for the compensation of backlash non linearity in [4]. Design and Analysis of Robust H infinity controller discussed in [5]. Switched Hybrid Speed Control of Elastic Systems with Backlash discussed in [6]. Estimation of Backlash with application to Automotive Power trains is described in [7], but in this methodology the control requires estimation of backlash size. An Adaptive Control Approach for Improving Control Systems with Unknown Backlash is presented in [8]. A Benchmark on Hybrid Control of a Mechanical System with Backlash is proposed in [9]. Speed Control of Torsional Drive Systems with Backlash described in [10].

A structure Doubling Algorithm for Discrete Time Algebraic Ricatti Equation suggested in [11]. Particle swarm optimization based proportional integral and derivative (PID) controller design for linear discrete time system using reduced order model is described in [12]. Tuning P-PI and PI-PI controllers for electrical servos in [13].Design Method for control system are described in [14].Newton Method for DARE when a close loop system has eigen values on the unit circle suggested in [15]. Generalized Ricatti Equation for the full and reduced order mixed norm H2/H∞ standard problem discussed in [16]

As compared to the strategies proposed above ([1]-[8]) and also with the performance of standard PID controller in the presence of disturbance and external noises, we designed the H infinity control for the system which guarantees the robustness and good performance. The advantages achieved by using this technique are, unaffected system output stability in the presence of external noises and disturbances, system achieved the preferred trajectory in quick time, high disturbance rejection, best transient response, absence of any limit cycle and steady state errors.

The arrangement of rest of the paper is as follows, Section II presents the system Model equations and a state space model of both modes. Section III presents discretization of System. In Section IV H infinity control is introduced. In Section V the proposed control strategy is described. Section VI shows the graphs through simulations. Section VII the result has been discussed on the basis of simulation graphs. Finally in Section VIII the conclusion has been illustrated with the future work.

II SYSTEM MODEL & EQUATIONS

The Mechanical system is operating in two modes

1. Contact Mode

When the contact between the two mechanical parts is established and the transmission of the torque takes place.

2. Backlash Mode

When the two mechanical parts are not in contact. In this case there is a gap between the transmission link of the motor and the load side.

In te real world there are number of mechanical systems in which backlash non linearity occurs. The model which has been selected here to analyze the robustness of H infinity controls was suggested in Nodrin [1].Actually the system is a two mass system. One mass representing motor side, other the load side and both are connected through gears. The System is as follows;

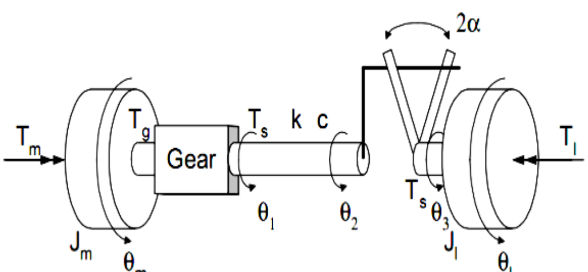

Fig.01 Two mass system with Backlash [1]

Jm	Motor Moment of inertia ($kg\ m^2$)	
Jl	Load Moment of inertia ($kg\ m^2$)	
Ts	Transmitted Shaft Torque (Nm)	
Tm	Motor Torque	(Nm)
Td	Load Torque disturbance (Nm)	
θd	Displacement Angle	(rad)
C	Inner damping coefficient of shaft	
Cm	Viscous motor Friction (Nm/(rad/sec))	
Cs	Viscous load Friction	(Nm/ (rad/sec))
ɤ	Turn ratio of the Gear Box	

A. Dynamic Equations

Dynamic equation of the figure 01 system is given below;

Equation at Motor Side

$J_m * \dot{\omega}_m = Tm - Ts/ɤ - Cm*\omega m$ (A)

Equation at Load Side

$J_l * \dot{\omega}_l = Ts - Td - C_l * \omega_l$ (B)

$\theta_d = \theta m/ɤ - \theta_l$ (Displacement Angle)

$\theta_b = \theta_d - \theta_s$ (Backlash Angle)

$\theta s = \theta d - \theta b$ (Shaft Angle)

$Ts = K*\theta s + C*\theta s$ (Contact Mode Shaft Torque)

$Ts = 0$ (Backlash Mode Shaft Torque)

Now by using equation (A), (B) & the values of shaft torque, State space model for both modes has been derived.

B. Contact Mode State Space Model

The discrete time LTI (Linear and time invariant) system is operating in two modes. The only single input of the system is the motor torque. There are multiple output of the system. Here in this case the four states that have been measured are motor angular velocity, Load angular velocity, motor angle and load angle. The fifth one backlash angle is zero in contact mode. So D matrix is over all zero.

$$\begin{bmatrix} \dot{\omega}_m \\ \dot{\omega}_l \\ \dot{\theta}_m \\ \dot{\theta}_l \\ \dot{\theta}_b \end{bmatrix} = \begin{bmatrix} -\frac{c_m + \frac{c}{\gamma^2}}{j_m} & \frac{c}{j_m \gamma} & -\frac{k}{j_m \gamma^2} & \frac{k}{j_m \gamma} & \frac{k}{j_m \gamma} \\ \frac{k}{j_l \gamma} & -\frac{c_l + c}{j_l} & \frac{k}{j_l \gamma} & -\frac{k}{j_l} & -\frac{k}{j_l} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_m \\ \omega_l \\ \theta_m \\ \theta_l \\ \theta_b \end{bmatrix} + \begin{bmatrix} \frac{1}{j_m} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} T_m$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ with state vector } \begin{bmatrix} \omega_m \\ \omega_l \\ \theta_m \\ \theta_l \\ \theta_b \end{bmatrix}$$

C. Backlash Mode State Space Model

In the backlash mode, all five states has been measured with the shaft torque is zero. Assuming that the disturbance torque is also zero in this case.C and D matrices will be the same.

$$\begin{bmatrix} \dot{\omega}_m \\ \dot{\omega}_l \\ \dot{\theta}_m \\ \dot{\theta}_l \\ \dot{\theta}_b \end{bmatrix} = \begin{bmatrix} \frac{-c_m}{j_m} & 0 & 0 & 0 & 0 \\ 0 & -\frac{c_l}{j_l} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \frac{1}{\gamma} & -1 & \frac{k}{c_\gamma} & \frac{-k}{c} & \frac{-k}{c} \end{bmatrix} \begin{bmatrix} \omega_m \\ \omega_l \\ \theta_m \\ \theta_l \\ \theta_b \end{bmatrix} + \begin{bmatrix} \frac{1}{j_m} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} T_m$$

$\omega_m, \omega_l, \theta_m, \theta_l, \theta_b$ are state variables.

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And D matrix will be the same as contact mode.

III DISCRETIZATION OF THE SYSTEM

In this paper, H infinity control technique has been applied to the discrete model of this system. As our system is of the form;

$\dot{X} = Ax + Bu$

Where A and B are time invariant matrices. The discrete system matrix is calculated by

$A_d = \exp(A*t)$

$B_d = \int \exp(A*(T-\delta))\, d\delta * B$

Cayley Hamilton Theorem has been used to calculate the state transition matrix for both system and input matrices. The sampling time of 0.1 sec has been use in the calculation of discrete time model matrices. The output Matrix C is same as for the continuous model i.e $C_d = C$.

IV H INFINITY CONTROL

The name H infinity derives from the fact that mathematically the problem may be set in the space $H\infty$ (named after the British mathematician G. H. Hardy), which consists of all bounded functions that are analytic in the right-half complex plane [12]. H infinity control methods are systematic and considered as most suitable to work for MIMO systems and allow in dealing with model uncertainty. In the last three decades the H infinity control problem for linear time invariant system has been studied extensively. During the instances, when the controller order is same as that of system order, the control problem has been solved by Algebraic Ricatti Equation approach [2].

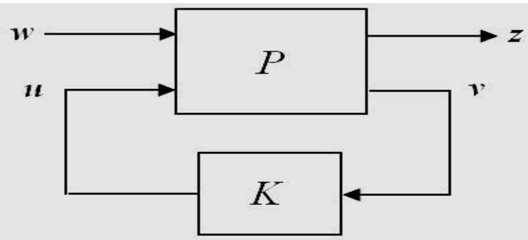

Fig.02 H Infinity Control Plant Model

The main objective for designing an H infinity controller is to guarantee robustness and good performance in the presence of disturbances.H infinity control is known for his strong useful mathematical foundation and the ability to include both classical and robust concepts together.

V PROPOSED CONTROL STRATEGY

The approach is basically a standard $H\infty$ problem, where by using the solution of Discrete Algebraic Ricatti Equation we have found the $H\infty$ control which guarantees disturbance rejection and explicitly deals with model uncertainties.The stabilization of an uncertain linear system by using state feedback control law has also been accomplished using a Discrete Algebraic Ricatti Equation.

A. DISCREETE ALGEBRAIC RICATTI EQUATION

$$A^*XA-X-A^*XB\,(B^*XB+R)^{-1}B^*XA+Q=0$$

A^* is the transpose of the discrete system matrix. B^* is the transpose of the input matrix B. System matrix A is of order n by n and B (input matrix) is m by n. Q and R are hermition matrices of size n by n & m by m respectively.

X is the solution matrix of order n by n which is to be found for which $(R+B^*XB)$ is invertible, such that the solution X is called admissible.

B. Conditions

1. $R>0$ & $Q\geq0$
2. (A, B) is Stabilizable.
3. All the eigen values of A must be on the unit circle. It is very difficult to find out the stabilizing solution of Discrete Algebraic Ricatti Equation when system matrix eigen values are on the or near to the unit circle.

Then there exists a matrix K such that eigen values of $(A+B*K)$ are inside the unit circle, i.e: stable, then $X\infty$ exists and $X>0$.

So the discrete time algebraic ricatti equation;

$$X\infty=A^*X\infty A+Q-(A^*X\infty B)*(R+B^*X\infty B)^{-1}*(B^*X\infty A)$$

Where $X\infty$ is the maximal hermite solution of discrete time algebraic Ricatti Equation.

C. H Infinity Control Law

The state feedback control law which has been designed for the H infinity control is given by;

$$u=-Kx+Nr \qquad \text{(C)}$$

Where x is the discrete time state of the mechanical system and K is the regulator gain defined by using maximal solution of discrete time algebraic ricatti equation. The expression for regulator gain K is as follows;

$$K=(R+B^*X\infty B)^{-1}\,B^*X\infty A.$$

The open loop regulator transfer function for this control is given by;

$$L=K*(z*I-A)^{-1}*B$$

The Sensitivity function S is defined as;

$$S=1/\,(1+K*(z*I-A)^{-1}*B)$$

So the final equation of the control law which has been used in the designing is given by;

$$u=-\,((R+B^*X\infty B)^{-1}\,B^*X\infty A)*x+Nr$$

The factor N is known as the Pre Scalar and is given by the following expression;

$$N=-1/\,(C*(A-B*K-I)^{-1}*B)$$

Where C is the output matrix and K is the gain matrix. The maximal hermit solution of Discrete Algebraic Ricatti Equation plays also an important role in the minimal factorization of realization.

D. Algorithm for Solution of DARE

There are number of algorithms used for the solution of DARE. The main focus is on this limitation that algorithm is intelligent enough to deal with any complex system plant and corresponding equations. To meet all these conditions, the NEWTON METHOD has been selected to find out the stabilizing solution of Discrete Time Algebraic Ricatti Equation. In this Method by solving a discrete time Lyapunov Equation (or a Stein equation) at each iteration, the solution of DARE has been obtained.

E. Convergence of Newton Method

The convergence of Newton Method is shown to either quadratic or linear with the common ratio of 1/2.

As we have discussed above that for the Backlash mode, there is the cases when motor loses its contact with the load and after this the load is neither controllable nor observable. The transmitted torque is zero and the system dynamics also changes. Here in this model, when a system operating in backlash mode, it has been found that the system uncontrollable parts has eigen values on the unit circle indicating clearly a marginally stable system. So the system has been partitioned into the uncontrollable and controllable parts. So we can say that in the backlash mode two states were controllable i.e. Motor Angular Speed and Motor Angle (Position) while other two states were uncontrollable i.e. Load Angular Speed and Load Angle (Position).

VI PID CONTROL ALGORTIHM

There are two steps involved in the proposed algorithm.

1. For the given higher order system, the equivalent reduced model can be obtained.
2. After obtaining the reduced order model, then design the PID control so that it can meet the desired specifications.

Simulation Parameters

$\alpha=0.029$, $Jm=0.3$, $Cm=0.1$, $Jl=140$, $Cl=5.6$, $C=120$, $K=10000$, $\gamma=12$ [4]. All parameters except Cm is adopted from Lagerberg [5].

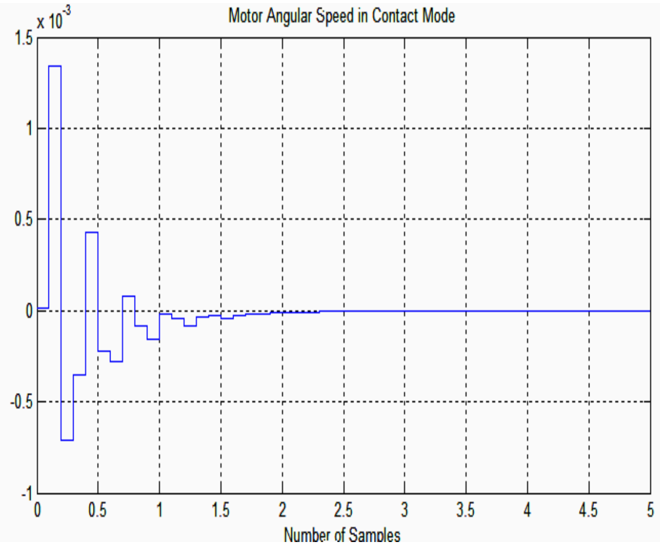

Fig.03 Stabilization of motor Angular Speed in Contact Mode.

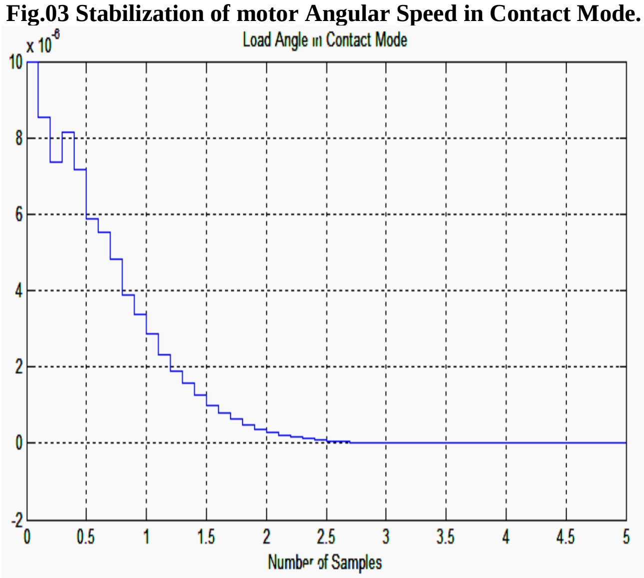

Fig.04 Stabilization of the Load Angle Speed in Contact Mode.

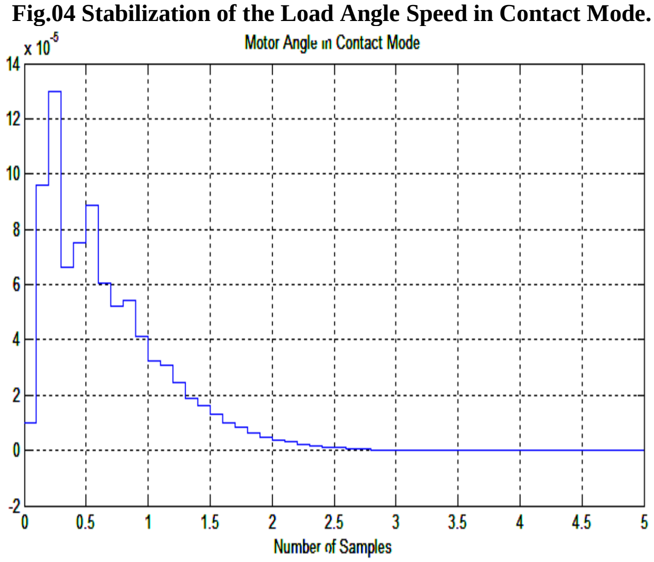

Fig.05 Stabilization of Motor Angle (Position) in Contact Mode.

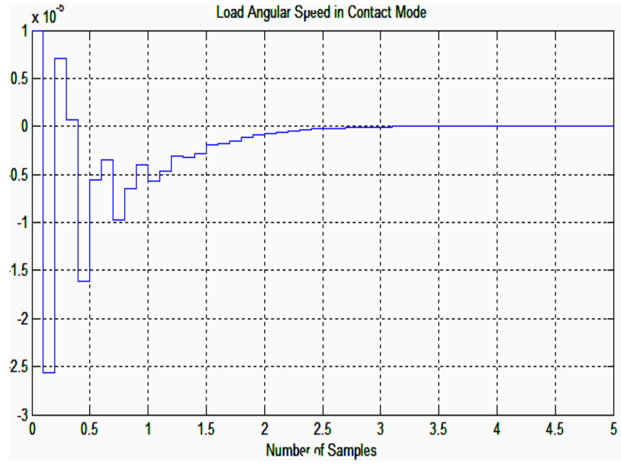

Fig.06 Stabilization of Load Angular Speed in Contact Mode By H infinity Control.

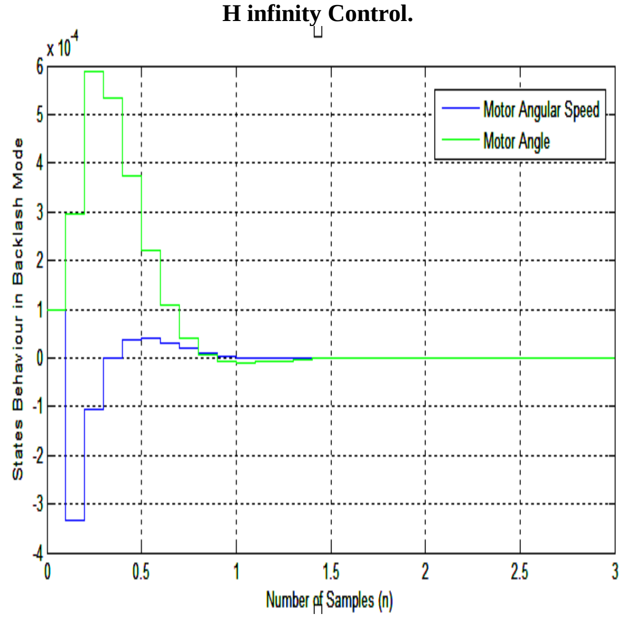

Fig.07 Stabilization of Backlash Mode States by H Infinity Control.

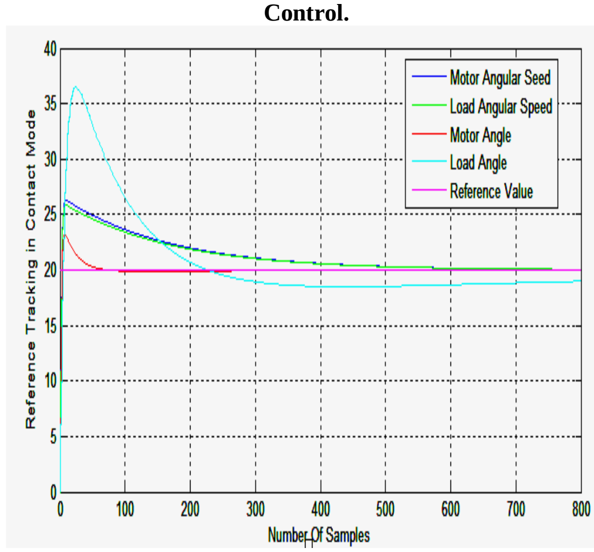

Fig.08 Stabilization of Contact Mode All States by PID control.

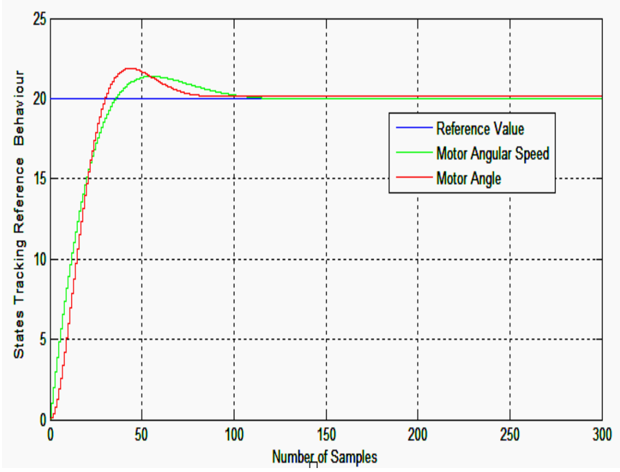

Fig.09 Reference Tracking of the Backlash Mode states by PID Control .

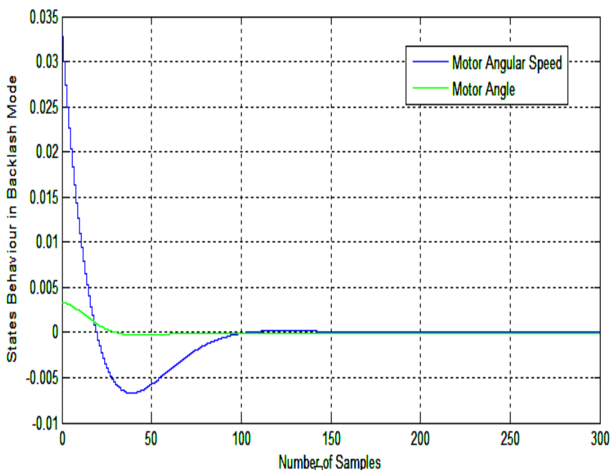

Fig 10.Stabilization of states in Backlash Mode by PID Control.

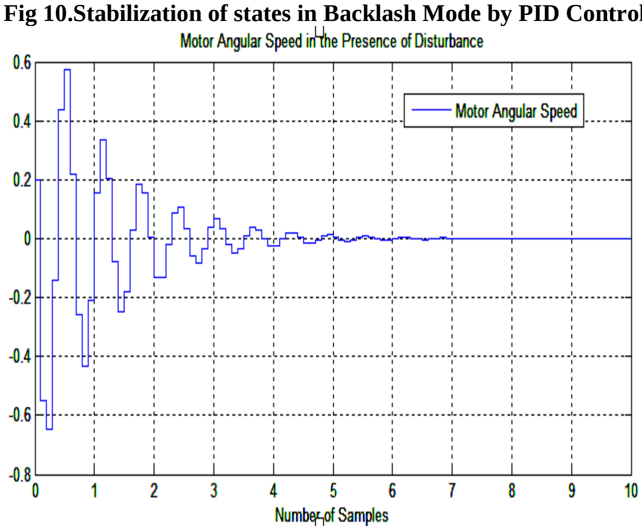

Fig 11 Stabilization of Motor Angular Speed in the presence of external disturbance.

VI SIMULATIONS

Gains in contact mode obtained from the DARE solution.
K1=1.3058, K2 =35.582, K3= -9.168,K4=131.5857.
Gains in Backlash Mode K1=0.2818, K2=1.1155
PID Controller Gains for the Contact Mode Kp=6.77, Ki=0.03, Kd=-0.55.PID Controller Gains for the Backlash Mode Kp=0.153, Ki=0.0599, Kd=0

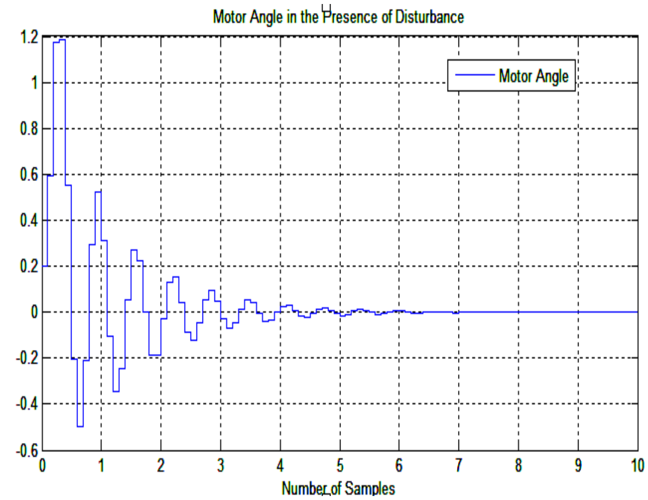

Fig.12 Motor Angle (Position) in presence of Disturbance.

VII RESULTS

The H infinity control has been designed for both the modes of contact i.e. Contact Mode & Backlash Mode. In Fig.03 the state motor angular speed in contact mode stabilization has been achieved with ease.The settling time of all the states is in between 0-5 sec and there is no steady state error in contact mode. In the Fig. 06 the load angular speed has been stabilized by the H infinity control with the minimum oscillation and zero steady state error. In fig.04 and fig .05 both motor angle and load angle states are stabilized. All the states within a very short time perfectly following the perfect trajectory and remains there for all times to come.

In the Fig.07 the backlash mode stabilization has been achieved within a very short sample time. Both the states, motor angular speed and motor angle decays to perfectly zero. The settling time of both the states is around 0.5 sec.

Fig.08 shows the performance of PID controller. The reference tracking of all the states in the contact mode has been shown in it. The stabilization and reference tracking in the backlash mode of the two states by PID controller has been shown in the Fig.09 and Fig.10.

As it has been mentioned earlier that H infinity control is robust enough to deal with the disturbance and completely reject it. Fig.11 shows the stabilization of motor angular Speed for the backlash mode in the presence of disturbance.Fig.12 shows the stabilization of motor position (Angle) in the presence of external disturbance.

VIII CONCLUSIONS

The infinity control design for both the modes of mechanical system has been designed and its performance been analyzed. Here the solution of discrete algebraic ricatti equation has been used to find out the H infinity state feedback control law. The suggested strategy requires the stabilizing solution of DARE on the bases of which controller has been designed.The proposed strategy is being investigated for the performance and position control in both modes of the model. Here the better results have been achieved in the presence of backlash, in terms of steady state error, robustness, good performance. We verify through simulations that H infinity control is providing high disturbance rejection and guaranting high stability as compared to the standard PID controller with backlash. The H infinity controller initiative the system efficiently to the preferred trajectory and quickly.

For future work, the improved design of individual controller in the case of backlash in any system is suggested.Research works on more enhance formulation for the close loop system stability and good performance for the systems with backlash is going on.

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